On the Cauchy Problem for a Weakly Dissipative Periodic Two-Component Dullin-Gottwald-Holm System

Haicui Lv, Yanli Wang, Jia Song

Haojing College of Shaanxi University of Science &Technology, 712046, China

Email: zhanglvfeicui@163.com

Keywords: Cauchy problem; Weak dissipation; Equation

Abstract: The local well-posedness of the weakly dissipative nonlinear shallow water wave equation is established. For different initial values, we obtain the global existence of the solution and the blow-up of the solution respectively. It is necessary to study the influence of dissipation term on the system solution. Due to the complexity of the equation itself, it is difficult to find a connection between each other. The abstract Cauhcy problem is the most significant application theme of the operator semi-group, so the two promote and grow together. The domain of the generator can be relaxed to the polynomial case without the need for dense and pre-solved estimates. This has broadly broadened its scope of application.

1. Introduction

Discussing the related properties of shallow water wave equation and revealing the propagation law of wave has great application value in explaining natural phenomena accurately and determining the attributes of physical materials [1]. Operator semigroups have been developing continuously for decades, which has formed a wide range of mathematical branches which have important applications in mathematics and engineering technology. Nonlinear Schrodinger equation (NLS) is one of the most important equations in mathematical physics. It describes many physical problems. Many papers have dealt with the existence, regularity and stability of its solutions [2]. Energy consumption is an inevitable phenomenon in nature, which is also common in the propagation of waves. Therefore, it is necessary to study the influence of the dissipative term on the system solution, which is in accordance with the natural law and is necessary [3]. Due to the complexity of the equation itself, it is difficult to find a connection between each other. It is almost necessary to invent special algorithms and apply novel techniques for each specific problem. The research on the correlation properties of shallow water wave equations has become one of the hot issues in the field of mathematical physics at home and abroad due to its wide application prospects in superelastic material mechanics.

That is, the operator semi-group is the most powerful tool in the study of abstract Cauchy problem, and the abstract Cauhcy problem is the most significant application subject of the operator semi-group, so the two are mutually promoted and grow together [4]. One of the reasons why the semigroup theory of operators has been developed so far is that people have introduced a wide variety of operator semigroups for various backgrounds [5]. Due to the penetration of nonlinear differential equations in many disciplines, it has become an extremely broad subject in the field of research. Due to the complexity of nonlinearity, many nonlinear systems have higher requirements on time and space dimensions and can only be limited to systems with lower dimensions [6]. The large-time behavior of solutions has attracted the attention of some researchers due to the in-depth study of infinite dimensional dynamic systems. It is impossible to establish a complete theoretical system in the few years that integral semigroups have been proposed, but the basic framework has been formed [7]. Generator domains need not be dense and estimates of predictors can be relaxed to polynomial cases. This has greatly broadened its scope of application.

  DOI: 10.25236/mmmce.2019.096

2. Cauchy Problem for Linear Hyperbolic Equations

Wave equations under weak nonlinear action can be extensively described by means of shallow water wave theory, since isolated waves cannot be found in the linear theory of smaller peaks [8]. Therefore, when dissipation occurs due to density non-uniformity during energy exchange, a weakly dissipative nonlinear wave equation is derived as in the kdv equation.

We study the following Cauchy problem for a weakly dissipative nonlinear shallow water wave equation with x being periodic and t being aperiodic:

$$w(x,y,d)=\exp(-(\frac{d_g}{r_g}+\frac{d_c}{r_c})) \tag{1}$$

Consider the abstract quasilinear evolution equation:

$$C(x,y,d^{(i)})=\sum_{(x,y)\in N(x,y)} w(x,y,d^{(i)})\cdot SelfAd(x,y,d^{(i)}) \tag{2}$$

In order to consider the weaker well-posedness of the abstract Cauhcy problem, a concept of distributed semigroups is proposed. If A is thick, then A generates an exponentially bounded distribution semigroup equivalent to A to generate an integral semigroup. and:

$$\begin{aligned} E_{LR}^{ij} &= \sqrt{\sum_{u=1}^{U}\phi_u^{ij}},\ j\in(1,M), i\in(1,H) \\ \phi_u^{ij} &= (S_{Lu}^{i}-S_{Ru}^{i})^2 \end{aligned} \tag{3}$$

After the input vector is determined, the distribution of variables is checked, and the data need to be transformed to facilitate the learning of the network [9]. For variables that are continuous, the commonly used normalization methods are:

$$I=\frac{24\pi^2A^2\gamma V^2}{\lambda^4}\left(\frac{n_1^2-n_2^2}{n_1^2+2n_2^2}\right)^2 \tag{4}$$

The network convergence time is greatly shortened, and the performance of the network is improved. The transformation method is:

$$\frac{\Pi}{c}=\frac{RT}{\overline{\overline{M}}_n}+A_2c \tag{5}$$

Comparisons and reference data are processed in dimensionless way. Meaning method was used to deal with:

$$D=\frac{RT}{L}\frac{1}{6\pi\eta r} \tag{6}$$

Under the interference of new factors, it is possible to produce new deviations, which need to continue to be controlled by the above methods:

$$r=k_2\theta_A\theta_B=\frac{k_2a_Aa_Bp_Ap_B}{(1+a_Ap_A+a_Bp_B)^2} \tag{7}$$

In the model, the intersection of internal, boundary and external subsets of objects is used to describe the topological relationship between two objects, which is expressed as:

$$r=\frac{k_2a_Ap_A}{1+a_Ap_A+a_Bp_B} \tag{8}$$

Due to the existence of air damping and the inhomogeneity of materials, the energy dissipation and attenuation of waves in propagation will be caused. Consider the abstract quasilinear evolution equation:

$$D_i = a + \sum_{j=1}^{n} b_j \ln(p_j) + r_i \ln(Y) + u \tag{9}$$

In the formula for calculating information gain, the information gain is:

$$u_2^{\beta} = xm_2 + (1-x)m_2 = m_2 \tag{10}$$

According to Cauchy's function idea, relevant data are obtained to describe in detail the way of calculating information gain:

$$F(x) = 1 / \sum (x_i - x_i^0)^2 \tag{11}$$

Normalize input variables and output variables. The following formula:

$$w(x, y, d) = \exp(-(\frac{d_g}{r_g} + \frac{d_c}{r_c})) \tag{12}$$

The nonholonomic constraint equation is:

$$E(x) = \sum_{j=1}^{n} E_j \tag{13}$$

If you design a 3-layer network, you have K input units and s output units. Then for the 3 layer network:

$$G_r(s) = \frac{K_r}{1 + T_r s} \tag{14}$$

Solitary wave phenomenon has become a significant physical phenomenon in shallow water waves. In physics, it is found that there are many nonlinear partial differential equations corresponding to mathematical models with such solutions [10]. Calculate the activation value of the output layer unit:

$$Q(u_{ij}) = \sum_{i=1}^{n} \underset{1 \le j \le m}{Max} \{ g_{ij}(T) \} \tag{15}$$

Samples are sent to the hidden layer unit by connection weight to generate new activation values of the hidden layer unit:

$$y_{f-n_m} = \sum_{i=1, i \ne n}^{N} \sum_{l=1}^{M} \sqrt{p_{i_l}} \boldsymbol{h}_{i,n_m}^{T} \boldsymbol{W}_{i,i_l} s_{i_l} \tag{16}$$

3. Weak Dissipative Periodic Two-Component System

Fixed T0> 0. In the maximal interval in which the solution y with the initial value y0 has the solution y, each element and two arguments in C([0， T];H1)∩ c1([0， T];L2) The function y∈ H1(Ω) is equivalent, where Ω = S × [0, T0], then:

$$\begin{aligned} \frac{d}{dt}\int \sqrt{\varepsilon_1 + y_+} &= \frac{1}{2}\int \frac{T_t}{\sqrt{\varepsilon_1 + y_+}} \chi(y>0) \\ &= -\int \frac{yu_x}{\sqrt{\varepsilon_1 + y_+}} \chi(y>0) - \int \frac{y_x u}{2\sqrt{e_1 + y_+}} \chi(y>0) - \int \frac{ey_x}{2\sqrt{e_1 + y_+}} \chi(y>0) \\ &= -\int u_x \sqrt{\varepsilon_1 + y_+} + \varepsilon_1 \int \frac{u_x}{\sqrt{e_{1+} y_+}} X(y>0) - \int \frac{uy_x + \varepsilon y_x}{2\sqrt{\varepsilon_1 + y_+}} X(y>0) \end{aligned} \tag{17}$$

For the first integral, the partial integrals are:

$$-\int u_x\sqrt{\varepsilon_1+y_+}=\int\frac{uy_x}{2\sqrt{\varepsilon_1+y_+}}X(y>0)+R(t,\varepsilon) \tag{18}$$

Where R(t, ε) is the value of $\sqrt{\varepsilon_1}$ u at S defined in the y> 0 interval. Noting also that:

$$\int\frac{\varepsilon y_x}{2\sqrt{\varepsilon_1+y_+}}X(y>0)=\varepsilon\int\nabla\sqrt{\varepsilon_1+y_+}=0 \tag{19}$$

There are:

$$\frac{d}{dt}\int\sqrt{e_1+y_+}=\varepsilon_1\int\frac{u_x}{\sqrt{\varepsilon_1+y_+}}+R(t,\varepsilon_1) \tag{20}$$

We know that:

$$\left|e_1\int\frac{u_x}{\sqrt{\varepsilon_1+y_+}}+R(t,\varepsilon_1)\right|\le 2\sqrt{\varepsilon_1}\int|u_x|\le\sqrt{\varepsilon_1}\left[1+\int\left(u^2+u_x{}^2\right)\right] \tag{21}$$

References

[1] Daneri, S. Cauchy Problem for Dissipative H?lder Solutions to the Incompressible Euler Equations. Communications in Mathematical Physics, 2014, 329(2):745-786.

[2] Mi Y, Mu C. On the solutions of a model equation for shallow water waves of moderate amplitude. Journal of Differential Equations, 2013, 255(8):2101-2129.

[3] Ming S, Lai S, Su Y. The Cauchy problem of a weakly dissipative shallow water equation. Applicable Analysis, 2018:1-16.

[4] Wang, Weiwei. Global weak solutions to the Cauchy problem of compressible Navier-Stokes-Vlasov-Fokker-Planck equations. Mathematical Methods in the Applied Sciences, 2016, 39(3):508-526.

[5] Jiang M, Xiang J. Asymptotic stability of traveling waves for a dissipative nonlinear evolution system. Acta Mathematica Scientia, 2015, 35(6):1325-1338.

[6] Jihong S, Mingyou Z, Yanbing Y, et al. Global Well-posedness of Cauchy Problem for Damped Multidimensional Generalized Boussinesq Equations. Acta Mathematica Scientia, 2014, 1648(1):55-108.

[7] Chugainova A P, Il'Ichev A T, Kulikovskii A G, et al. Problem of arbitrary discontinuity disintegration for the generalized Hopf equation: selection conditions for a unique solution. IMA Journal of Applied Mathematics, 2018, 82(3):496-525.

[8] Blanc X, Ducomet B, Ne?asová, S. On some singular limits in damped radiation hydrodynamics. Journal of Hyperbolic Differential Equations, 2016, 13(02):249-271.

[9] Giesselmann J, Pryer T. Reduced relative entropy techniques for a posteriori analysis of multiphase problems in elastodynamics. Bit Numerical Mathematics, 2014, 56(1):1-29.

[10] Kan H, Yang X. A two-dimensional glimm type scheme on cauchy problem of two-dimensional scalar conservation law. Acta Mathematica Scientia, 2017, 37(1):1-25.